(Byshkin et al. 2016)

Byshkin, Maksym, Alex Stivala, Antonietta Mira, Rolf Krause, Garry Robins, and Alessandro Lomi. 2016. “Auxiliary Parameter MCMC for Exponential Random Graph Models.” *Journal of Statistical Physics* 165 (4): 740–54. https://doi.org/10.1007/s10955-016-1650-5.

Below is direct quotes from the paper:   
Byshkin et al. (2016) explain setting auxiliary parameter MCMC algorithm for ERGMs of undirected networks. MCMC simulation is necessary for estimating the expectation of graphs as a given set of model parameters. Several MCMC samplers that differ in the proposal distribution have been suggested for ERGMs. In a basic sampler, a dyad, i and j, are selected uniformly at random. The corresponding tie variable \x\_i\_j is toggled: delete the tie between these actors if it exists and add a tie if it is absent. Another sampler is the fixed density (FD) sampler, which modifies the basic sampler by fixing the number of times equal to that in the observed network. The FD sampler may be described as follows: select randomly one null dyad and one non-null dyad. The proposal is to toggle the values of both these dyads simultaneously. This proposal does not modify the number of network ties. If, at the starting point, the network has observed network ties, then all the graphs generated by this sampler have a constant number of ties. The FD algorithm samples not from the probability distribution \Pr(X=x) but from the probability distribution \Pr`(X=x)~ \delta\_L(x),\_L\_obs

Another popular MCMC sampler for ERGMs is the so-called “TNT” (Tie-No-Tie) sampler. Large social networks are typically sparse and this sampler is designed to alleviate problems that may arise due to sparseness. As in the Basic sampler, only one dyad is toggled at each simulation step of the TNT sampler, but rather than selecting a dyad to toggle uniformly at random, the algorithm first tosses a fair coin to decide whether to toggle a null (filling move) or a non-null dyad (deleting move). Once the move type has been determined, the dyad to be toggled is chosen uniformly at random among the available null (for a filling move) or non-null dyads (for a deleting move). With the TNT sampler, null dyads are chosen with probability 1/2 (instead of the proportion of non-null dyads as in Basic sampler, which is close to 1 for sparse networks).

Byshkin et al. (2016) propose the improved fixed density (IFD) MCMC sampler for ERGMs. Like the FD sampler, it decreases the number of allowed network states, however it does so without worsening the mixing of Markov chains. This is accomplished by introducing an auxiliary parameter and a computationally simple method for finding its value. At the same time the proposal distribution of IFD sampler is very similar to that of the TNT sampler making it useful also for sparse networks.

If the MCMC simulation converges to its equilibrium stationary distribution, then further MCMC transitions do not modify this distribution. In this case, the average properties of the system under study are constant. In the case of ERGMs such properties are values of statistics zA(x). Consider transitions that modify the number of ties L(x) by ±1 (as e.g. in the Basic or TNT samplers). The stationary distribution is reached only if the average probability that MCMC transitions increase L(x)equals the average probability that MCMC transitions decrease L(x).